CHAPTER FIVE

FUNCTIONS AND ITS ASSOCIATED SIMPLIFICATION

Simplification:

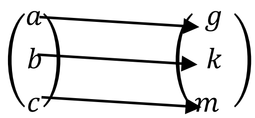
- Let x and y be two sets. When each number of the set x is associated or related to only one member of the set *y*, then such a relation is known as a function from x to y.
- This is written as f: x → *y* and read as "the function from the set x to the set y or by the equation y = f(*x*).
- The set x is known as the domain and the set y is known as the co-domain or the images.
- The word function emphasizes the idea of the dependence of one quality on another. For example, let f be the mapping which is defined by f: $x \rightarrow 2x+1$, which can be written as $y = 2x + 1$. We say that y is a function of *x* which means that y depends on x
- The variable x is called the independent variable, and y is called the dependent variable. The type of relation between *x* and y is called a functional relation. Each of the following defines the same set.

1) F: $\{x \rightarrow 2x - 1, x \in N\}$.
2) F = $\{(x,y): y = 2x - 1, x \in N\}$.
3) F = $\{x, 2x - 1: x \in N\}$.
4) Y = $2x - 1, x \in N$.
5) F(x) = $2x - 1, x \in N$.

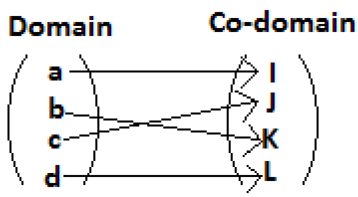
A function (or mapping) is therefore the relation between the elements of two sets, which are the domain and the co-domain, such that each element within the domain is associated or related to only one element in the co-domain.

Example (1)

Domain Co-domain



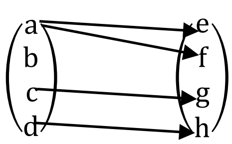
Example (2)



This is also a function, since each member of the mdomain is associated with only one member of the co-domain.

Example (3)

Domain Co-domain



This is not a function, for the first member of the domain i.e a, is associated with two members of the co-domain.

(Q1.) Given that F(x) = 2x+1, evaluated the following:

f(2) (b.) f(4) (c.) f(-3)

(d) f(-1) (e.) 2f(x) (f.) 5f(x).

<u>Soln.</u>

F(x) = 2x+1 =>

a.F(2) = 2(2)+1 = 4+1 = 5.

b. F(4) = 2(4)+1 = 8+1 = 9.

c. F(-3) = 2(-3)+1 = -6+1 = -5.

d. $F(-1) = 2(-1)+1 = -2+1 = -1$.

e. Since $f(x) = (2x+1) \Rightarrow 2f(x) = 2(2x+1) = 4x+2$.

f. $5f(x) = 5(2x+1) = 10x +5$.

N/B: $F(x) = 2x + 1$ can be written as $F(x) = (2x + 1)$ or $F(x) = 1(2x+1)$.

(Q2.) If $g(x) = 3x – 1$, evaluate the following:

a.$g(-1)$ b.) $g(-2)$ c.) $g(^1/_2)$

d.) $3g(x) +1$ e.) $4 g(x) – 2$

f.) $-2g(x)+2$ g.) $-3g(x) -3$.

<u>Soln.</u>

$g(x) = 3x – 1 =>$

a.$g(-1) = 3(-1) -1 = -3 – 1 = -4$.

b. $g(-2) = 3(-2) -1 = - 6 – 1 = -7$.

c. $g(^1/_2) = 3(^1/_2) -1 = 3 \text{ x } ^1/_2 – 1 = 1.5 – 1 = 0.5$.

d. $g(x) = 3x – 1 => 3g(x) + 1 = 3(3x-1) + 1 = 9x – 3 +1 = 9x – 2$.

e. $g(x) = 3x – 1 => 4 g(x) -2 = 4(3x - 1) -2 = (12x - 4) -2 = 12x - 4 - 2 = 12x – 6$.

Q3. Given that $f(x) = 3x + 2$ and $g(x) = -4x – 2$, evaluate the following.

a.) (i) $f(-1)$ (ii) $f(-2)$

b.) (i) $g(-1)$ (ii) $g(-2)$ (iii) $g(2)$

c.) (i) $f(x) + g(x)$ (ii) $f(x) – g(x)$

d.) (i) $2f(x) + 3$ e.) $3f(x) – 2$

f.) $g(x) – f(x)$

Soln.

a.) $f(x) = 3x + 2 =>$

(i) $f(1) = 3(1) + 2 = 3 + 2 = 5$.

(ii) $f(-2) = 3(-2) + 2 = -6 + 2 = -4$.

b.) $g(x) = -4x - 2 \Rightarrow$

(i) $g(-1) = -4(-1) - 2 = 4 - 2 = 2$.

(ii) $g(-2) = -4(-2) - 2 = 8 - 2 = 6$.

(iii) $g(2) = -4(2) - 2 = -8 - 2 = -10$.

c.) (i) $f(x) + g(x) = (3x + 2) + (-4x - 2)$

$= 3x + 2 - 4x - 2 = 3x - 4x + 2 - 2$

$= -x + 0 = -x$

(ii) $f(x) - g(x) = (3x + 2) - (-4x - 2)$

$= 3x + 2 + 4x + 2 = 3x + 4x + 2 + 2$

$= 7x + 4$.

d.) $2f(x) + 3 = 2(3x + 2) + 3$

$= 6x + 4 + 3 = 6x + 7$.

e.) $3f(x) - 2 = 3(3x + 2) - 2 = 9x + 6 - 2$

$= 9x + 4$.

f.) $g(x) - f(x) = (-4x - 2) - (3x + 2)$

$= -4x - 2 - 3x - 2 = -4x - 3x - 2 - 2$

$= -7x - 4$.

Q4. A function $f: x \to 3x+2$, is defined on the set x

$= \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$.

a. Find the images of the following:

i. -3   ii. -1   iii. 2   iv. 5

b. Find the value of x for which

i. $F(x) = 8$ ii. $F(x) = 11$ iii. $F(x) = -4$

Soln.

a) i. $F: x \rightarrow 3x+2$ and for the image of -3, put $x = -3 \Rightarrow f(x) = 3x+2 \Rightarrow f(x) = 3(-3) +2 = -9+2 = -7$.

ii. For the image of -1, put $x = -1$. From $f(x) \rightarrow 3x+2 \Rightarrow f(x) = 3(-1)+2 = -3+2 = -1$.

iii. For the image of 2, put $x = 2$. From $f(x) = 3x+2 \Rightarrow f(x) = 3(2) +2 \Rightarrow f(x) = 6+2 = 8$.

iv. For the image of 5 put $x = 5$. $F(x) = 3x+2 = 3(5) +2 = 15+2 = 17$.

b) i. $F(x) = 3x+2$. If $f(x) = 8 \Rightarrow 8 = 3x+2 \Rightarrow 8 - 2 = 3x, \Rightarrow 6 = 3x \Rightarrow 3x = 6, \Rightarrow x = {}^{6}/_{2} \Rightarrow x = 3$.

ii. $F(x) = 3x+2$ and if $f(x) = 11 \Rightarrow 11 = 3x+2, \Rightarrow 11 - 2 = 3x$

$\Rightarrow 9 = 3x, \Rightarrow 3x = 9 \Rightarrow x = 9/3, \Rightarrow x = 3$.

iii. $F(x) = 3x+2$ and if $f(x) = -4 \Rightarrow -4 = 3x + 2, \Rightarrow -4 -2 = 3x \Rightarrow 3x = -6, \Rightarrow x = {}^{-6}/_{3} \Rightarrow x = -2$.

Q5. A function $f: x \rightarrow 8x+1$ is defined on the set x

$= \{-1, 0, 2, 3, 4, 5\}$

a. Find the images of -1 and 3.

b. Find the value of x for which $f(x) = 7$.

Soln.

$F(x) = 8x+1$.

(a) For the image of -1, put $x = -1 \Rightarrow f(x) = 8(-1) +1 = -8+1 = -7$.

For the image of 3, put x = 3 $\Rightarrow$f(x) = 8(3) +1 = 24+1 = 25.

b. F(x) = 8x+1. If f(x) = 7 => 7 = 8x+1 => 7-1 = 8x, $\Rightarrow$ 6 = 8x => 8x = 6, => x = $^{6}/_{8}$ = 0.75.

Q6. A function f(x) = $\frac{5x-2}{2x+1}$ is defined on the set of real numbers.

a. Determine the images of the following:

i. -2 ii. -1 iii. 2 iv. 4

b. Evaluate the following:

i. f(3) ii.f(6)

c. Find the value of x for which f(x) is undefined.

Soln.

a. f(x) = $\frac{5x - 2}{2x + 1}$

i. For the image of -2, put x = -2 => f(x) = $\frac{5(-2) - 2}{2(-2) + 1}$

= $\frac{-10 - 2}{-4+1}$

= $\frac{-12}{-3}$ = 4

ii. For the image of -1, put x = -1 =>f(x) = $\frac{5(-1)-2}{2(-1)+1}$

= $\frac{-5-2}{-2+1} = \frac{-7}{-1} = 7.$

iii. For the image of 2, put x = 2 => f(x) = $\frac{5(2) - 2}{2(2) + 1}$

= $\frac{10-2}{4+1}$

= $^{8}/_{5}$ = 1.6

b. i. f(x) = $\frac{5x - 2}{2x + 1}$ => f(3) = $\frac{5(3) - 2}{2(3) + 1}$

= $\underline{15 - 2}$ = $\underline{13}$ = 1.8.

$6 + 1 \quad 7$

ii. $f(x) = \frac{5x - 2}{2x + 1}$, $\Rightarrow f(6) = \frac{5(6) - 2}{2(6) + 1}$

$= \frac{30 - 2}{12 + 1} = \frac{28}{13} = 2.1.$

C For the value of x for which the function is undefined, put the down part to be equal to zero and solve for x.

i.e. $2x + 1 = 0 \Rightarrow 2x = 0 - 1$,

$\Rightarrow 2x = -1 \Rightarrow x = \frac{-1}{2} = -0.5.$

$\therefore$ The function is undefined when $x = -0.5$.